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Computer Modelinq of an Oil Flooded Single Screw Air Compressor

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COMPUTER MODELING OF AN OIL FLOODED SINGLE

SCREW AIR COMPRESSOR

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ABSTRACT

A computer model of a single screw oil-flooded low pressure air compressor was developed which predicts the internal conditions based on certain empirical values. A characteristic thread technique is used for steady state operation. A polytropic process is used and the model will iterate for the polytropic coefficient to best correlate internal predictions with empirical values.

The geometric variables and relationships used to describe the general cylindrical mainrotor-planar gaterotor single screw mechanism are developed. The control volumes and their corresponding governing thermodynamic relationships are defined. The equations used to model the leakage for the various paths are described. The results of the model are compared to the empirical data and the validity of the assumptions used to reduce the general equations is discussed.

INTRODUCTION

The single-screw mechanism is a rotary positive-displacement concept being investigated for use as compressors and pumps in naval applications. There is particular interest in establishing the feasibility of a water-flooded high pressure air compressor configuration. A first step in investigating this feasibility was to develop a model which quantifies leakage losses as a function of assumed fluid dynamic and thermodynamic characteristics. The configuration modeled was a cylindrical mainrotor/planar gaterotor version illustrated in Figure 1. The computer model was then applied to the evaluation of an oil-flooded, 966 KPa, 1800 rpm compressor manufactured by Chicago Pneumatic Tool Company.

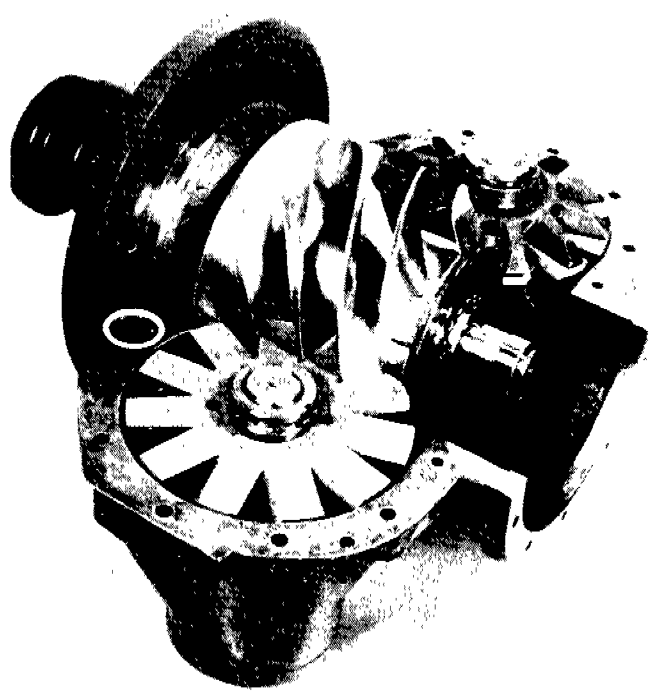

Figure 1
Single-Screw Air Compressor

This computer model predicts the internal conditions of the compression process, pressures, temperatures and flow through the leakage paths, by using empirically determined external values of pressures, temperatures and flowrates measured at the intake and discharge of the compressor. The model is divided into three fundamental categories; kinematics, thermodynamics and the fluid dynamics through the leakage paths.

The single screw compressor gets its name from the one mainrotor. There is a thread cut into the mainrotor so that a meshing tooth on the planar side rotor, referred to as the gaterotor, can travel through the mainrotor thread. The

* Based on MSME Thesis [1]** while a graduate student at the Ray W. Herrick Laboratories, Purdue University, West Lafayette, IN 47907.
** The number of brackets refers to the list of references.

volume between the mainrotor threads and ahead of the gaterotor tooth is then the compression chamber.

The compression process begins by filling the thread with suction air. The suction flow is axial through the open end of the main rotor. When the thread is filled with air, a gaterotor tooth meshes with the mainrotor, and closes the suction port. The oil injection begins and continues until the thread passes the injection orifice. The oil acts as a coolant, lubricant and a sealant. The compression of the air continues until the mainrotor thread uncovers a radial discharge port located in the housing. As the gaterotor tooth sweeps out the mainrotor thread, compressing the air, the thread is filling on the bottom side of the gaterotor tooth. The gaterotor tooth on the other side of the mainrotor then engages the same mainrotor thread, and the process repeats.

Kinematics

The geometric variables necessary to define a single screw compressor and some typical values as shown in Figure (2) are:

NM = 6 = number of threads in main rotor
NS = 11 = number of gaterotor teeth
RM = 8.738 cm = mainrotor radius
RS = 8.575 cm = gaterotor outer radius
RI = 5.011 cm = gaterotor inner radius
W = 2.570 cm = width of gaterotor tooth
θsc = -22 deg = gaterotor angle at suction closure
DV = 3.6 = dry volume ratio - thread volume at suction closure/thread volume as discharge port opens.

The swept volume history is calculated by determining the engaged area of the gaterotor tooth and multiplying by the distance the centroid is rotated about the mainrotor axis. The area of the gaterotor tooth is found, for example, by defining the four corner points, shown in Figure (2), by the following:

where $\gamma = 2\ \mathrm{SIN}^{-1}\ (W/(2\ RS))$
$Z(1) = RI\ \mathrm{TAN}\ (\theta) + W/2\ \mathrm{COS}\ (\theta)$
$R(1) = RM$
$Z(2) = RS\ \mathrm{SIN}\ (\theta + \gamma)$
$R(2) = RM + RI - RS\ \mathrm{COS}\ (\theta + \gamma)$
$Z(3) = RS\ \mathrm{SIN}\ (\theta - \gamma)$
$R(3) = RM + RI - RS\ \mathrm{COS}\ (\theta - \gamma)$
$Z(4) = RI\ \mathrm{TAN}\ (\theta) - W/2\ \mathrm{COS}\ (\theta)$
$R(4) = RM$

The area and centroid $\overline{R}$ are then evaluated by the following by Wojciechowski [2].

$$\mathrm{Area} = \sum_{i=1}^{n} \left(z_{i+1} - z_i\right)\left(R_{i+1} + R_i\right)/2$$

$$\overline{R} = \frac{1}{\mathrm{Area}} \sum_{i=1}^{n} \left[\left(z_{i+1} - z_i\right)/8\right]\left[\left(R_{i+1} + R_i\right)^2 + \left(R_{i+1} - R_i\right)^2/3\right]$$

The volume swept in the mainrotor increment $\Delta\alpha$ is
$\mathrm{Vol} = \mathrm{Area}\ \overline{R}\Delta\alpha$

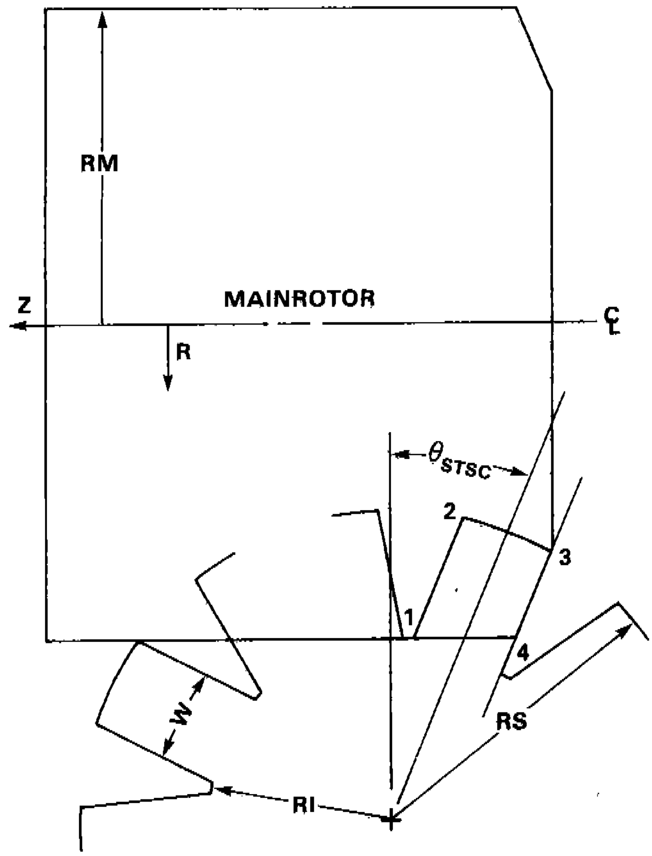

Figure 2
Geometric Variables

The increment volume versus mainrotor angle is plotted on Figure (3) and the points where the suction port closes and the discharge port opens are indicated.

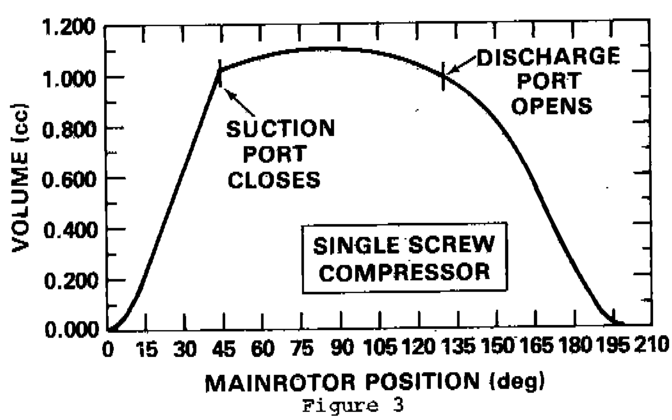

Figure 3
Incremental Volume

THERMODYNAMIC MODELING

The modeling of the compression process is subdivided into the Suction-Closure Process, the Closed-Compression Process and the Discharge Process. The assumptions used in the analysis are:

(1) The air behaves as an ideal gas.
(2) The properties within the control volume are uniform.
(3) Gravitational and kinetic energies are negligible.
(4) Inlet air is preheated to a uniform temperature before entering thread.
(5) Frictional losses across the inlet port are negligible.
(6) Pressures and temperatures in the suction and discharge plenums are constant.
(7) The values predicted for one thread will be the same as the values in the other threads when the appropriate phase angle is accounted for.
(8) There will not be any air leaking across the leakage paths within the control volumes.
(9) Any oil that leaves the closed compression chamber either directly into the trailing thread or into an area connected to the suction chamber will be assumed to go into the trailing thread.
(10) There will not be any backflow of oil into the control volume when the discharge port opens and the discharge pressure is higher than the pressure in closed compression chamber.
(11) The heat transferred between the air in the compression chamber and the oil will be negligible.
(12) The heat transfer between the air and the oil passing through the discharge port will be assumed to continue until they reach thermal equilibrium.

SUCTION-CLOSURE PROCESS

The boundaries of the control volume are shown on a planar representation of the outside of the mainrotor in Figure (4). The suction-closure process models the entrance of the gaterotor tooth into the thread. From Figure (4) it can be seen that the size of the control volume reduces from the beginning to the end of the process. Therefore, as the gaterotor tooth moves into the thread reducing the volume, air will flow out of the thread through the inlet port. As the mainrotor thread gets closer to closing on the gaterotor tooth the suction port area reduces. This causes a slight pressurization of the air in the control volume and it is modeled to get an accurate value of the mass in the control volume.

The governing energy equation used to model this process is the first law of thermodynamics. The effect of suction air preheat was evaluated experimentally and the average temperature of the air entering the thread is assumed constant from this basis. Because the air in the control volume is being pressurized there will be no mass of air entering the thread. The general equation can be simplified to:

$$-\int W_{cv} - h_{out}\, dm_{out} = dU_{cv}$$

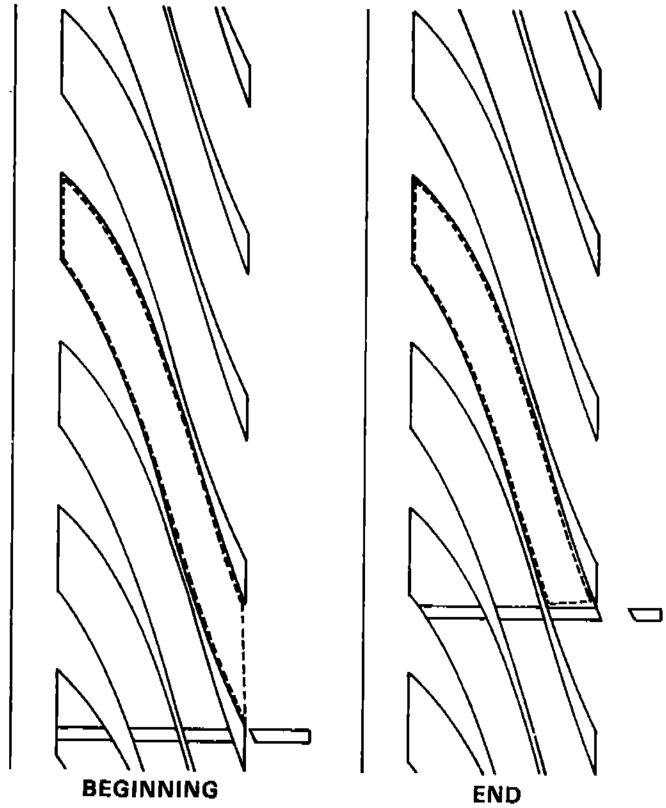

Figure 4
Boundaries of Suction Closure Control Volume

where:

$\int W_{cv}$ = differential quantity of work done to the control volume

h_{out} = enthalpy of gas in control volume

dm_{out} = mass rate of air flow discharging from control volume

dU_{cv} = rate of change of internal energy in control volume

Rewriting the work, the internal energy, assuming constant specific heats and simplifying, the pressure in the control volume, Pcv, can be written:

$$P_{cv} = \frac{T_{cv}\, R\, dm_{cv}}{dV_{cv}} \qquad (1)$$

where:

T_{cv} = temperature in control volume

R = gas constant

dm_{cv} = mass rate of air flow out of the control volume

dV_{cv} = rate of change of volume in control volume

The mass of air flow out of the control volume is evaluated by applying Bernoulli's equation.

The rate of change of the volume is a function of the geometry and of the oil flow rate in and out of the control volume via the leakage paths. Equation (1) is solved by incrementing through the cycle and by employing a predictor-corrector scheme. The new pressure is predicted, P_p, by reducing the volume by the geometric volume increment between the time steps. This is written:

$$P_p = \frac{T_{cv} \, Rm_n}{V_{n+g}}$$

where:

m_n = mass in control volume at beginning of increment.

V_{n+g} = available volume in control at end of increment due to geometry.

This predicted pressure is then used to evaluate the mass flow of air and the mass flow of oil during the time increment. Due to the incompressibility of the oil, the mass flow of oil can be related to a volume change. ΔP, ρ, and P are quantities that are required to evaluate the mass flow rates that change during the increment. To account for this, the average of the values at the beginning and end of the increment are used. The relationship for corrected pressure is written:

$$P_c = \frac{\dot{T}_{cv} \, Rm_c}{V_c}$$

Where:

P_c = the corrected pressure at n+1.

V_c = the corrected volume at n+1.

m_c = the corrected mass of air at n+1.

If the difference between P_p and P_c is above a user set tolerance limit, the procedure is repeated by setting P_p equal to P_c until convergence is reached.

CLOSED COMPRESSION PROCESS

The closed compression process begins when the beginning of the mainrotor trailing thread meets the gaterotor trailing flank, thereby closing the inlet port. The closed compression process ends when the leading thread of the mainrotor overlaps the trailing edge of the discharge port. The boundaries of the control volume begin with the end boundary for the suction closure process shown in Figure (4) and are subsequently defined by the edges of the mainrotor thread, the casing and the gaterotor. The gaterotor and mainrotor positions at the end of the closed compression process is shown in Figure (5). The only openings in the boundaries of the control volume are the leakage paths. By assumption, only oil passes through these, so the mass of air in the control volume is constant.

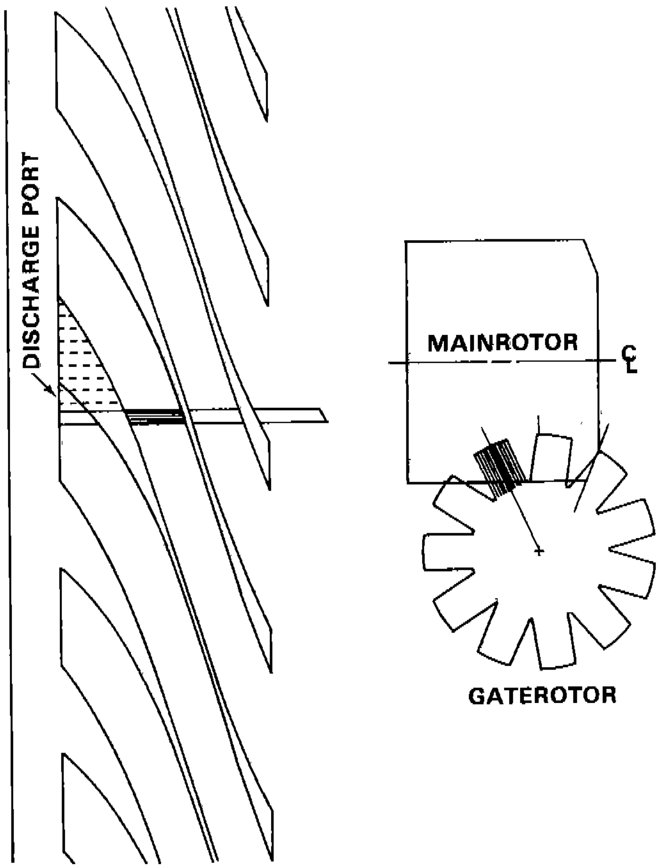

Figure 5
End of Closed Compression Process

There is assumed to be no heat transfer between the air and the oil while in the control volume. This is supported by the idea that the process occurs too quickly for the oil to absorb a significant amount of heat from the compressed air. The governing energy equation is the first law of thermodynamics for a closed process. The amount of heat transfer from the air being compressed is not known, therefore the polytropic process relationship is used.

The value of the polytropic exponent, n, will be determined by matching known temperature points from data on a compressor with values predicted from this process relationship. The temperature of the air oil mixture is known after discharge from the thread. The value of n will be determined by assuming a value for n and incrementing through the process. At the end of the closed compression process, the temperature that the mass of air and oil, at their associated temperatures, contained in the control volume would reach if allowed to go to equilibrium will be computed. This temperature will then be compared to the known temperature of the mixture at discharge. If the predicted temperature is not within a tolerance limit of the known point, a secant method interpolation is used to predict a new value of n. This procedure is repeated until the predicted temperature is within the tolerance limit set.

The pressure of the control volume is determined with a similar predictor-corrector scheme as in the discussion of the suction closure process. In this case, the polytropic relation will be used and the predicted pressure is written as:

$$P_p = P_n(V_n/V_{n+g})^n$$

Where:

- P_n = The pressure at the beginning of the increment.
- V_n = Available volume of control volume at beginning of ncrement.
- V_{n+g} = Available volume of control volume at end of increment due to geometry.

The predicted pressure is used to evaluate the mass flow of oil through each of the leakage paths. The volume available to the air at the end of the increment is equal to the original volume plus the geometric increment plus the volume due to oil flow. This is written:

$$P_c = P_n\ (V_n/V_{n+1})^n$$

Where V_{n+1} is the volume available to the air at the end of the increment. Obviously, the pressure varies during the increment. The mass flows of oil need the pressure of the control volume for the calculation. To compensate for the changing pressure an average pressure during the time increment is used by the mass flows of oil.

The predictor-corrector procedure is repeated until the difference between the predicted and corrected pressure is below a tolerance limit. When the limit has been satisfied, the temperature of the air is found from the ideal gas relation. This is written:

$$T_{n+1} = \frac{P_{n+1}\ V_{n+1}\ R}{m_{cv}}$$

Where:

- T_{n+1} = Air temperature at the end of the increment.
- P_{n+1} = Pressure in control volume at the end of the increment.

DISCHARGE PROCESS

The discharge process begins when the leading thread of the main rotor overlaps the trailing edge of the discharge port. The discharge process ends when the gaterotor trailing flank disengages from the mainrotor. The discharge port opens the control volume to the discharge plenum. The discharge process will be considered to be the combination of closed volume compression, and flow either in or out of a rigid vessel. When the pressure in the control volume is less than the discharge plenum pressure, only air will be considered to flow into the control volume. Also, the discharge plenum pressure and temperature will be assumed to be constant and the heat transferred to the surroundings from flow through the discharge port will be assumed to be negligible.

For the closed volume compression, the polytropic relationships described in the closed compression process will be used to generate predicted values of the pressure and temperature in the control volume at the end of the increment due to the gometric volume change. The polytropic exponent n will be the value that was iterated for during the closed compression process. Having compressed the air, the control volume boundary is conceptually opened allowing flow through the discharge port. The governing relationship for both flow into and out of the control volume is the first law of thermodynamics. A predictor/corrector scheme similar to those previously described was developed to evaluate the relationships.

The case when the corrected pressure is less than the discharge pressure is physically unrealistic and was modeled by adjusting the predicted pressure so that the pressure at the end of the increment is the discharge pressure. This was done because it makes it possible to calculate a pressure gradient across the discharge port. This is not only used to evaluate the mass of air that leaves the control volume, but it is needed to evaluate the mass of oil that leaves the control volume. With this approach, the mass of air and mass of oil can be tracked during the discharge process.

LEAKAGE MODELING

From the original set of assumptions, only oil will be considered to pass through the leakage paths that are on the boundary of the control volume. This will have a significant effect on the results of this model and is based on the premise that the oil is directed to the leakage paths by the rotating motion of the components. There are 9 leakage paths through which fluid can flow either into or out of the control volume. These are:

(1) Gaterotor tip
(2) Gaterotor leading flank
(3) Gaterotor leading flank blowhole
(4) Gaterotor trailing flank
(5) Gaterotor trailing flank blowhole
(6) Gaterotor window
(7) Mainrotor leading thread land
(8) Mainrotor trailing thread land
(9) Mainrotor discharge end band

These paths are identified on Figure (6) and (7). These paths were divided into three categories for analysis: (1) constant clearance gap, (2) variable clearance gap and (3) orifice type.

Flow paths that have very short throttling lengths will be considered to be orifices. The criteria for this will be (L/D) <2.5; where L is the

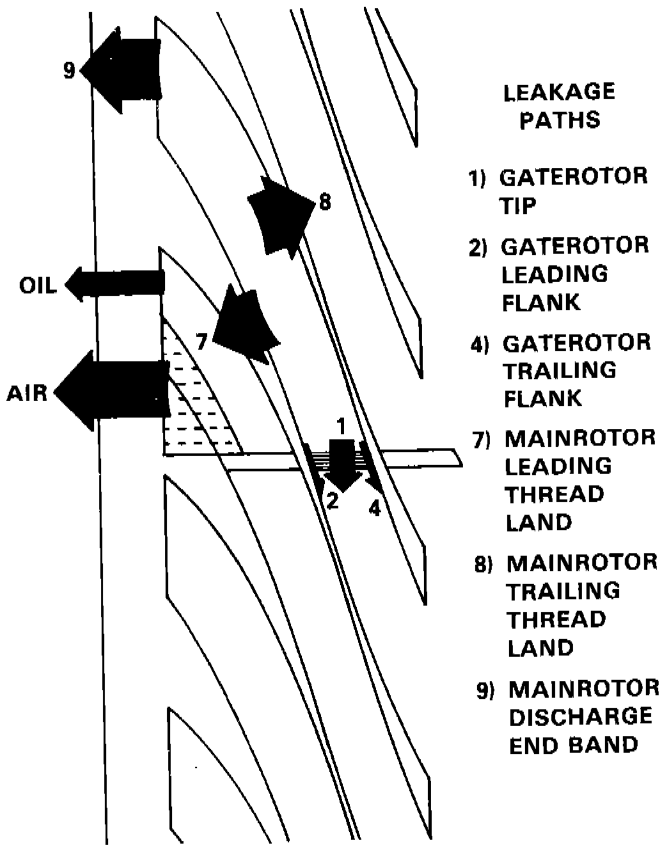

Figure 6
Leakage Paths - Mainrotor View

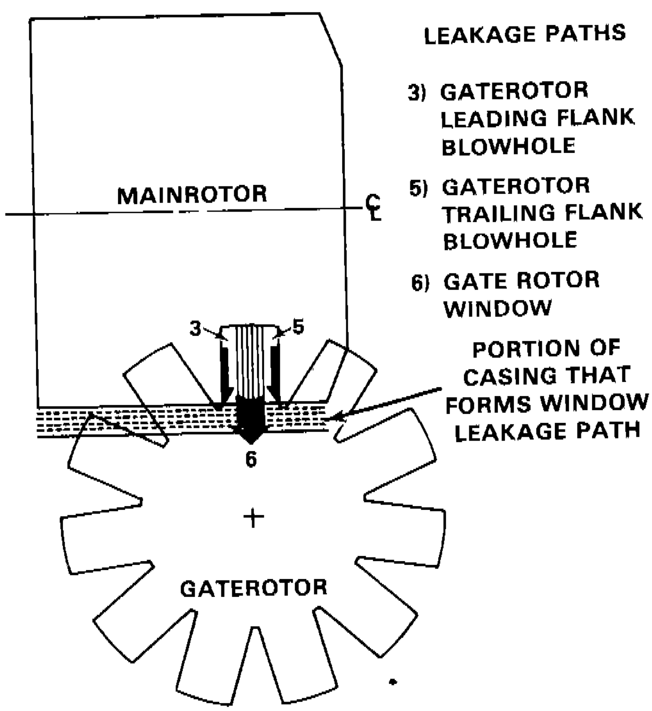

Figure 7
Leakage Paths-Gaterotor View

throttling length parallel to the flow and D is the hydraulic diameter. [5] The governing equation for flow through the orifice paths will be Bernoulli's equation. The governing equation for incompressible flow not considered to be orifice flow will be Darcy's formula. This is:

$$h_L = \frac{\Delta P}{\rho} = \frac{V^2}{2} f \frac{L}{D} = K \frac{V^2}{2}$$

Where:

f = friction factor.
h_L = head loss across path.
K = resistance coefficient.
P = pressure drop across path.
V = velocity of flow through path.
ρ = mass density of flowing fluid.

The losses of a fluid flowing through a particular path are generally a result of viscous effects and a result of changes in speed and direction of the flow path. To adjust the predicted flow rate to match the flow rate obtained from operating data, correction factors are typically used. For this study the correction factors will only adjust the nonviscous losses. The viscous losses are quite well defined, especially in the laminar flow region, [6] and will not be considered to be effected by the speed at which the parts are moving relative to each other. The nonviscous effects will, however, be affected by the speed of the parts. It is recognized that the correction factor that will be used will be an average value. The relative speed between the parts will change during the cycle. By using an average correction factor, the error in the calculated oil flow through the particular path will have a negligible effect on the predicted pressure history in the control volume. For the leakage paths where the gap clearance varies, Darcy's equation was appropriately modified and intergated through the throttling path.

DISCHARGE PORT

There is a leakage path at the discharge port that is not within the control volumes previously defined. Figure (6) also shows the direction and location of the flow. The fluid leaking through this path will be assumed to be air. This is consistent with the assumption that there will not be any backflow of oil into the control volume from the discharge plenum. The flow will be assumed to be one dimensional and isentropic. The governing equation used is derived by Soedel in [7], and the result is written as:

$$m = A\, P_{DP} \sqrt{\frac{2K}{(K-1)\, RT_{DP}}} \sqrt{r^{2/K} - r^{\frac{K+1}{K}}} \qquad (2)$$

Where:

A = Cross sectional area.
P_{DP} = Pressure, discharge plenum.
T_{DP} = Temperature, discharge plenum.
r = Pressure downstream/pressure upstream (limited by critical pressure ratio)

The assumption that air is leaking through the path, does not account for the liquid in the path. Referring to Figure (6) and ignoring the other threads for a moment, it can be seen that oil flows between the mainrotor and housing before the thread reaches the discharge port. Having reached the discharge port, air flows between the housing and the mainrotor. But before the air can flow through the gap, according to the governing equation, the oil must be pushed out of the gap. As a result of this argument, a correction factor will be applied to equation (2). This correction factor will be determined by correlating the predicted values with the available empirical values.

RESULTS

The correction factors that modify the effect of the nonviscous losses were adjusted so that the predicted changes in volumetric efficiencies due to known changes in gaterotor tip path and the window path gap clearances approximated the changes measured in the empirical data. The correction factors for the other paths were estimated using the similarity of the relative velocities to the two paths that had an empirical basis.

Although the available empirical data does not evaluate changes in the clearance of the discharge gap, the data available is sufficient to evaluate what value to assign to the discharge port correction factor. The effect on the volumetric efficiency due to the suction air preheat and the various leakage paths have each been adjusted in the model to yield the response measured by the empirical data. The only leakage path unaccounted for is the effect due to the discharge port leakage. Therefore, the discharge port leakage-path correction factor will be adjusted until the model predicts the same value as the empirical data at a particular point. The factor was evaluated to be 0.154, using a discharge pressure of 965 KPa as the comparison point. An interpretation of this value is that for approximately 85% of the time, the air flow through the discharge port leakage path is blocked by oil.

The value of the polytropic exponent that the model evaluated in order to try to meet the air-oil bulk discharge temperature was 1.4. The predicted bulk air-oil discharge temperature was 62 C., whereas the empirical value was 65 C. The model was limited to a maximum polytropic exponent of 1.4 and the lower air-oil bulk discharge temperature indicates that heat is transferred into the control volume. This heat gain is most likely a result of an assumption in the model that the oil that leaks between control volumes stays at the oil injection temperature. The oil will increase in temperature just by friction, which would be enough to allow the predicted bulk temperature to reach the empirical bulk temperature.

Figure (8) shows the comparison of the empirical data and the model values for changes in the gaterotor tip clearance. Figure (9) presents the relative contribution of each of the factors that effect the volumetric efficiency of the compressor.

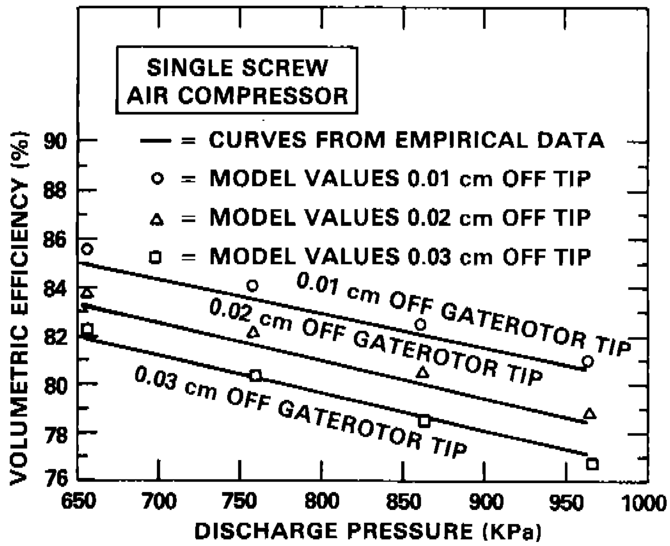

Figure 8
Gaterotor Volumetric Efficiency for Changes in Gaterotor Tip Clearance - Model Values

CONCLUSIONS

From the results of the comparison between the computer model and the empirical data, the following conclusions can be drawn:

(1) A computer model can be constructed to predict the internal performance of a single screw compressor by adjusting correction factors in the model to produce empirical external data.

(2) The fluid that passes through the leakage paths in the boundary of the control volume can be modeled as being only oil, if a correction is applied to the nonviscous effects.

(3) The viscous losses for oil flow through nonparallel wall leakage paths can be approximated by integrating Darcy's equation.

(4) The closed compression process can be modeled as an isentropic process.

(5) There is insignificant heat transfer between the air and oil during the compression process. The majority of the heat transfer between the air and oil occurs in the discharge port.

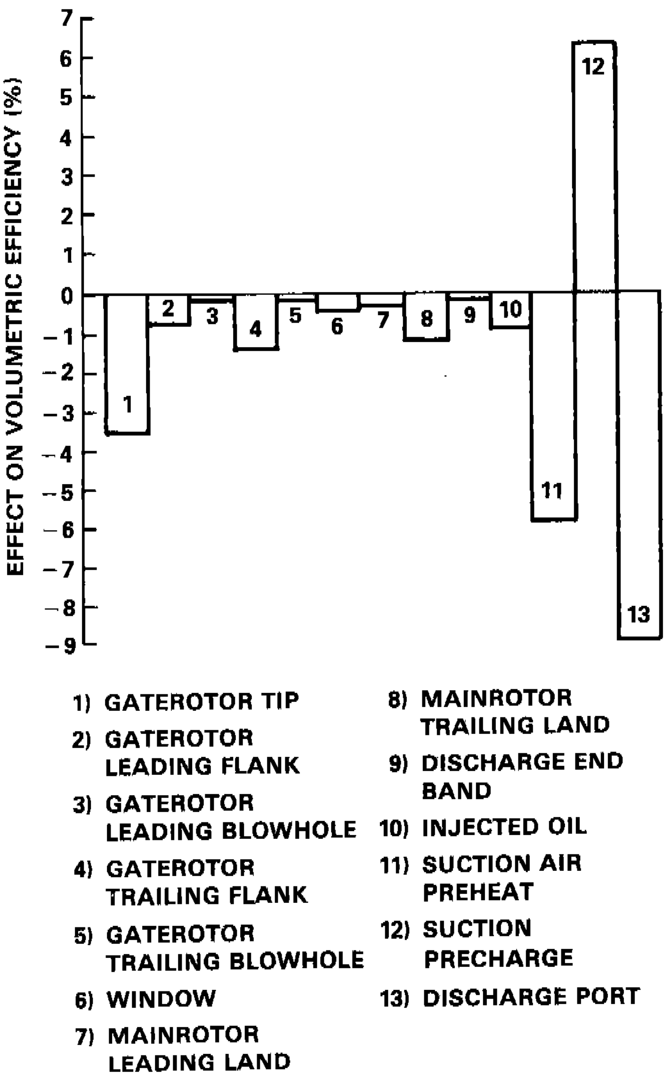

Figure 9
Comparison of Individual Effects on the Volumetric Efficiency - Model Values

REFERENCES:

1. Bein, Thomas W., A Computer Model of a Single Screw Air Compressor, MSME Thesis, Purdue University, 1980

2. Wojciechowski, Felix, Machine Design, Jan 22, 1976, p. 105.

3. Hseigh, Jui S., Principles of Thermodynamics, 1975, Scripta, Washington, DC.

4. Fox, R.W. and McDonald, A.T., Introduction to Fluid Mechanics, 2nd edition, 1978, John Wiley, New York.

5. The Engineering Division, Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper 410, 1979, The Crane Company, New York.

6. Stepanoff, A.J., Centrifugal and Axial Flow Pumps, 1948, John Wiley, New York.

7. Soedel, Werner, Introduction to Computer Simulation of Positive Displacement Compressors, Ray W. Herrick Laboratories, Purdue University, July 1972.